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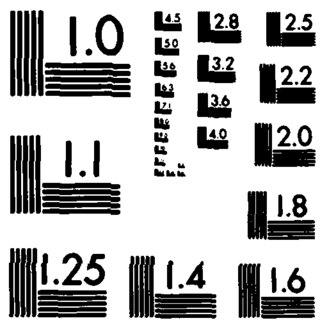

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AFOSR-82-0277

Evangelos A. Coutsias

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Nonneutral plasmas confined by conducting walls exhibit, among other things, steady oscillations of the type referred to as virtual cathode oscillations.

In previous work we studied the instability of steady plasma flow between two conducting grids in one dimension (Sullivan and Coutsias, 1981; Coutsias and Sullivan, 1983). In the later reference we used ideas of bifurcation theory to numerically demonstrate the existence of an oscillatory state for neutral beams which was not previously established.

What was still lacking was a method of dealing with the analytical problems created when a cold beam develops regions of multivalued flow. The main theoretical difficulties are present in a simplified one-dimensional model in which monoenergetic electrons are injected through a grid in a semi-infinite space and return to the grid due to the image-charge attraction.

For the dynamical description of the evolution of a monoenergetic stream we used a simplified version of the Vlasov equation for the electron distribution function. Our equation is essentially the Hamilton-Jacobi equation for the action of an electron, subject to the self-consistent electric field, i.e., the field caused by the other electrons plus any externally imposed field.

It is proved that the stream will evolve continuously. We view the particle velocity profile $u = u(x,t)$ as a surface in (u,x,t) space. Then, the situation is similar to that of evolving ray surfaces in geometric optics (Arnol'd, 1978). Simple geometrical arguments show that under realistic assumptions about the self-consistent potential, the surface will develop folds, first appearing as cusps (Fig. 1). These shapes are generic in two-dimensional surfaces and will

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persist under perturbation. The self consistent field can be thought of as such a perturbation (at least for short enough times) so that this qualitative picture is relevant to our problem.

From this picture it is clear that at positions of folding the electron density will have a singularity of strength

$$\rho_{fold} \sim (x-x_f)^{-1/2}$$

while at cusp points ("first" formation of virtual cathode), the singularity is stronger

$$\rho_{cusp} \sim (x-x_c)^{-2/3} .$$

Interaction with the self consistent potential will, in general not affect this qualitative picture. However it does modify it in some exceptional cases and this has consequences for the stability properties of steady folding flows. From a mathematical point of view this is what makes this problem interesting: the self-consistent electric field, if it is computed locally on each stream has a swallowtail when the density rpofile has a fold (Fig. 1). *One needs to generalize the existing* results on the singularities of Lagrangian manifolds to account for derivative singularities of the field under which they evolve. Such generalization do not exist at present (Arnold, 1984).

For higher dimensions even stronger singularities become possible. It is seen that, at least for the singularities of one-dimensional flows, no charge resides there: the integral of the density in an interval of vanishing size about the singularity is zero.

From the time of realization that a singularity is present in the charge density describing a folding cold flow, its nature and significance remained rather controversial and misunderstood (Pierce, 1945; Poukey and Rostoker, 1971; Miller, 1982; Kadish, 1984). It is clear from our work that the singularity is a generic feature of cold beam descriptions, very much as the singularity appearing in the study of wave equations by the method of geometric optics. There the flow is really nonsingular, due to finite wavelength of the light. Here, a small thermal spread in the incoming distribution causes the turning point of the flow to be distributed over space, smearing out the singularity.

In the study of shock waves, viscocity effects give a finite thickness to the shock transition region, smoothing out the density discontinuity. Nevertheless, weak solutions play a very important simplifying role in gas dynamics. Using the cold flow approximation and discussing the evolution of fronts of monoenergetic particles can play a similar role in the study of cold nonneutral plasmas.

In our previous work we approached the study of virtual cathode oscillations by a model involving a monoenergetic stream of particles in steady motion under the influence of a strong opposing field. It is easy to see that this problem has a steady two-stream solution but deciding about its stability can be very hard. The above results suggest that stability of such flows is best addressed by considering the geometry of perturbations of the velocity profile: more traditional perturbative procedures cannot handle the density singularity and would fail.

To develop some of the ideas necessary for this work a simpler problem was considered: in Coutsias and McIver (1984) (App. 1) the quantum mechanical problem of scattering of electrons by a standing (laser) light wave was studied by the quasi-classical approximation. This problem is simpler than the problem of an electron beam in a semi-infinite space in that it only treats the effects on the electron motion from a time-dependent external field. Nevertheless, many of the qualitative features are similar in both cases.

An important methodological similarity exists between the quantum problem and the case of an electron beam with small thermal spread. In the first case (in the quasi-classical limit) quantum effects are dominant at caustics and prevent a density singularity. In the second the same thing will be accomplished in practice by the thermal spread, present in any physical situation.

Thus the cold beam approximation will be valid away from caustics, while near caustic we will have a boundary layer in which thermal spread is important. In analogy to the relation between maximum amplitude at a caustic and wavelength that we can get in geometric optics, here we can find a relation between maximum charge density and temperature T of the beam (for small T). Our results in that direction include an estimate of the lowering of the space charge limit of an electron beam due to temperature (Coutsias, 1984).

At this point, the question of analytically estimating the parameters of the oscillatory state has not yet been resolved. The difficulties involved are due to the nonlocal character of the oscillations. A model ignoring the interaction between the two streams was found to be exactly solvable and helped to illuminate the delay nature of the oscillation. It is found that for sufficiently low values of the incoming charge density this model has steady state. As the density, and consequently the ratio (imposed field)/(self field) is increased, the model leads to a periodic profile for the caustic which exhibits further bifurcation for even larger currents. The relevance of the higher bifurcations, even for this simplified model, is doubtful.

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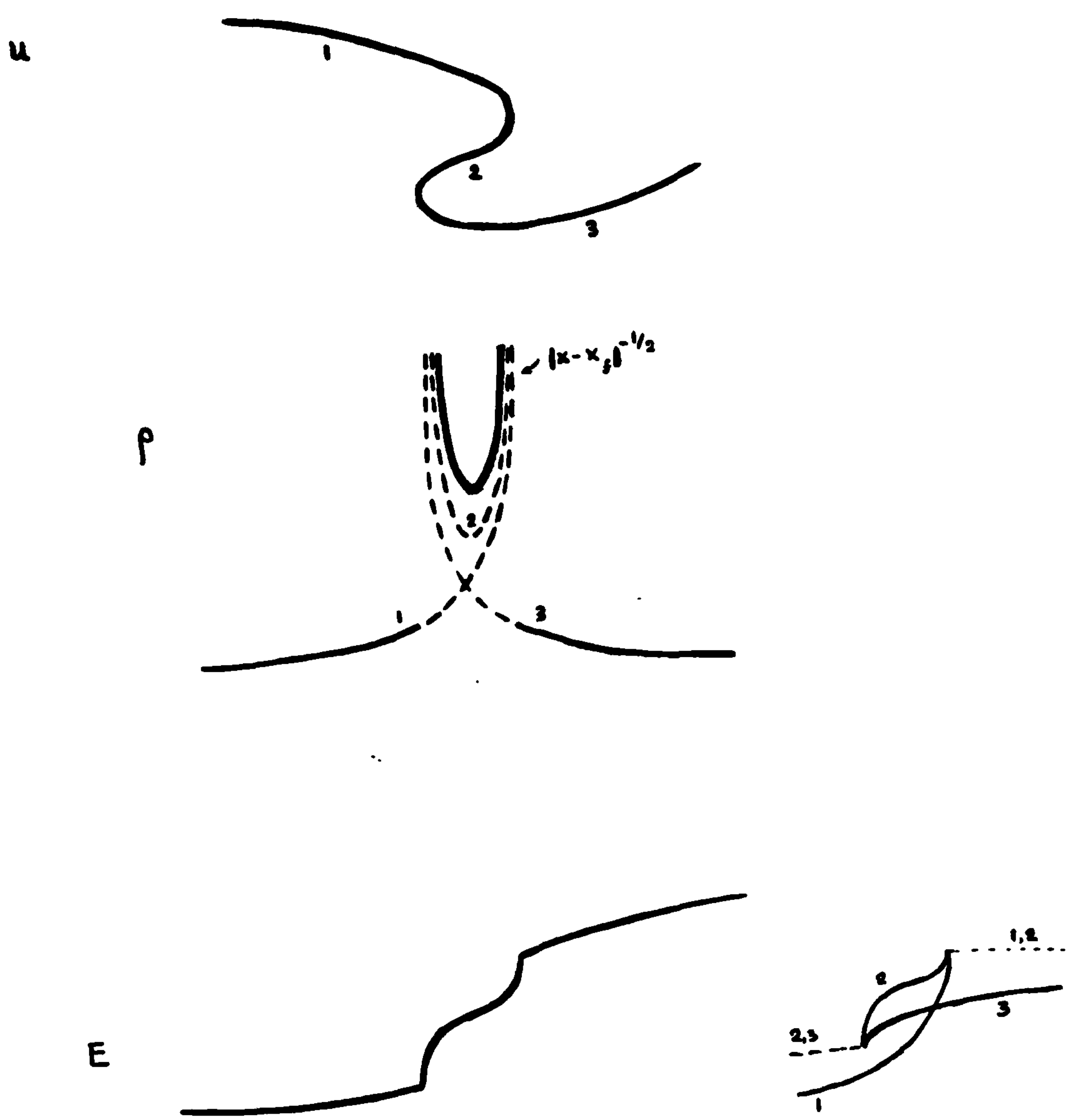

Fig. I

Velocity, (u), density (p) and electric field (E) as functions of position for a folded stream. The field is continuous since the charge singularity is integrable. (insert shows field due to each stream).

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